Computer Graphics

LECTURE 11

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Last Class

- Geometric Objects
  - Vector Space
  - Affine Space

Today's Agenda

- Geometric Objects
  - Vector Space
  - Affine Space
  - Basic Geometries

Notation

- Greek letters $\alpha, \beta, \gamma, \ldots$ denote scalars;
- uppercase letters $P, Q, R, \ldots$ denote points;
- lowercase letters $u, v, w, \ldots$ denote vectors.

Point

- Specifies a location in 3D space and is represented by three coordinates as (x, y, z).
  - In 2D space, however, only two coordinates will be needed (x,y,z)
  - The point is infinitely small and
  - Does not possess any shape

(x, y)

2D - space

Vector

- Represented by 3 coordinates and Specifies
  - Magnitude ($||V|| = \sqrt{dx^2 + dy^2 + dz^2}$)
  - Direction
  - Has no location

(dx, dy, dz)

Vector Space

- Vectors define a vector space
  - They support vector addition
    - Commutative and associative
    - Possess identity and inverse
  - They support scalar multiplication
    - Associative, distributive
    - Possess identity

Affine Spaces

- Vector spaces lack position and distance
  - They have magnitude and direction but no location
- Combine the point and vector primitives
  - Permits describing vectors relative to a common location
- A point and three vectors define a 3-D coordinate system
- Point-point subtraction yields a vector

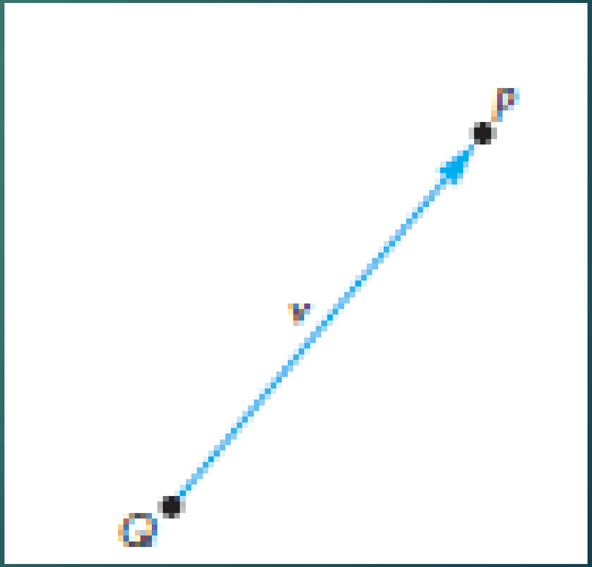
Point – Vector Operations

- Point – point subtraction yields

$$v = P - Q$$

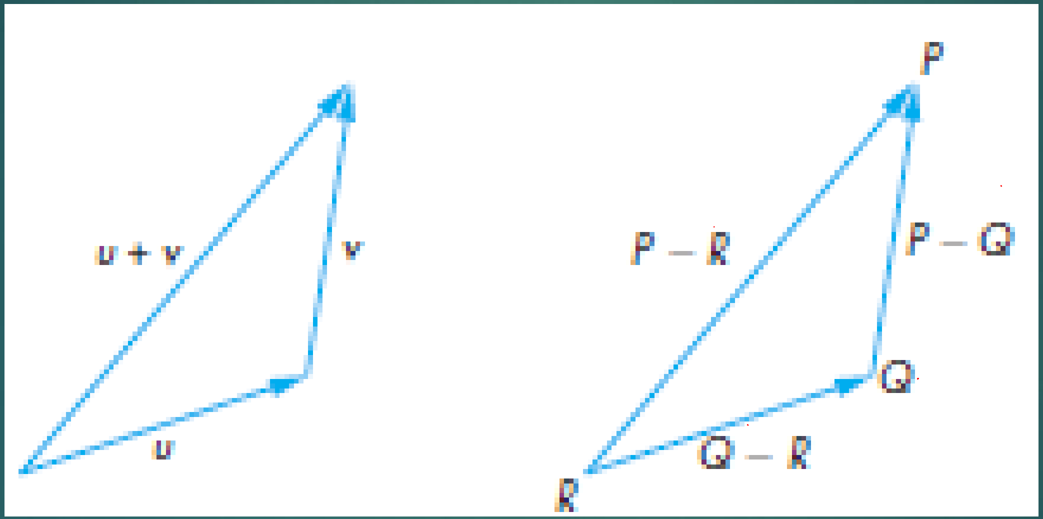
- Point – Vector addition yields

$$P = Q + v$$



Vector Addition

- We can also use this visualization to show that for any three points *P*, *Q*, and *R*

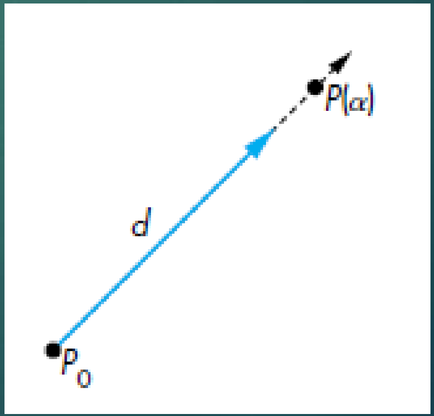


Lines

- Consider all points of the form

$$P(a) = P_0 + a\,\mathbf{d}$$

  - where P_0 is an arbitrary point d is an arbitrary vector and a is a scalar (that can vary over a range)
  - This relation can be interpreted as the set of all points that pass through P_0 in the direction of the vector d



Euclidean Affine Spaces

- Allows to compute distance and angles
  - Dot product: The dot product of two vectors is a scalar.
  - Let v_1 and v_2 be two vectors

(INNER PRODUCT)

$v_1 = (x_1, y_1, z_1)$.

$v_2 = (x_2, y_2, z_2)$.

$v_1 \cdot v_2 = (x_1, y_1, z_1) \cdot (x_2, y_2, z_2)$.

$= x_1 x_2 + y_1 y_2 + z_1 z_2$.

Dot Products

- Used to compute length (magnitude) of the vector

$$V = (x, y, z)$$

$$V \cdot V = (x \cdot x + y \cdot y + z \cdot z)$$

$$= x^2 + y^2 + z^2$$

$$\|V\| = \sqrt{V \cdot V} = \sqrt{x^2 + y^2 + z^2}$$

Dot Products

- Normalization (finding unit vector)

$$\frac{v}{\|v\|} = \hat{v}$$

$v = (2, 3, 2)$

$\|v\| = \sqrt{4+9+4}$

$= \sqrt{17}$

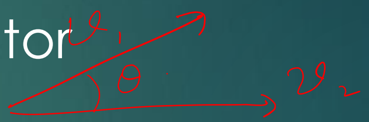
Normalized vector.

$$u = \frac{v}{\|v\|} = \left(\frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right)$$

$$\|u\| = \sqrt{\left(\frac{4}{17} + \frac{9}{17} + \frac{4}{17}\right)} = \sqrt{\frac{4+9+4}{17}} = \sqrt{\frac{17}{17}} = \sqrt{1} = 1$$

Dot Products

- Computing angle between two vector



v_1

v_2

$$v_1 \cdot v_2 = v_1 v_2 \cos\theta$$

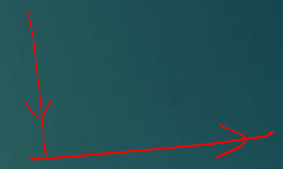
$$\cos(\theta) = \frac{v_1 \cdot v_2}{|v_1 \cdot v_2|} = \frac{v_1}{\|v_1\|} \cdot \frac{v_2}{\|v_2\|}$$

$$\theta = \cos^{-1}\left(\frac{v_1 \cdot v_2}{\|v_1\| . \|v_2\|}\right)$$

Dot Products

- Checking for orthogonality

$$v_1 \cdot v_2 = \underline{v_1 v_2} \cos\theta$$

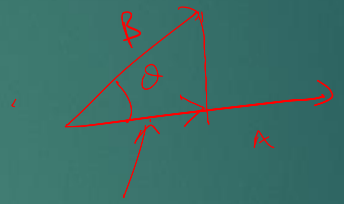


$$\theta = \pi/2 = 90^\circ$$

Dot product is zero if the vectors are orthogonal.

Dot Products

- Finding projection of a vector along another vector



$AB\cos\theta$

Dot Products

- Dot product is commutative and distributive

C .. $v \cdot u = u \cdot v$.

D $\rightarrow$ $u \cdot (v + w) = u \cdot v + u \cdot w$.

Summary

- Point
- Line
- Vector
- Dot Product

References

- Fundamentals of Computer Graphics Third Edition by Peter Shirley and Steve Marschner
- Interactive Computer Graphics, A Top-down Approach with OpenGL (Sixth Edition) by Edward Angel.